Course Outcomes Guide (COG)

Course/Program Title: Math 208, Linear Algebra **Date:** May 2013

Course/Program Team: Christopher Lewis

Course Outcomes:

Upon successful completion of this course students will be able to:

1) Use computational techniques and algebraic skills essential for the study of systems of linear equations, matrix algebra, vector spaces, eigenvalues and eigenvectors, orthogonality and diagonalization. (Computational and Algebraic Skills).
2) Use visualization, spatial reasoning, as well as geometric properties and strategies to model, solve problems, and view solutions, especially in R^2 and R^3, as well as conceptually extend these results to higher dimensions. (Geometric Skills).
3) Critically analyze and construct mathematical arguments that relate to the study of introductory linear algebra. (Proof and Reasoning).
4) Use technology, where appropriate, to enhance and facilitate mathematical understanding, as well as an aid in solving problems and presenting solutions (Technological Skills).
5) Communicate and understand mathematical statements, ideas and results, both verbally and in writing, with the correct use of mathematical definitions, terminology and symbolism (Communication Skills).
6) Work collaboratively with peers and instructors to acquire mathematical understanding and to formulate and solve problems and present solutions (Collaborative Skills).

General Education Outcomes:
Upon successful completion of this course students will be able to:
1. Apply mathematical methods involving arithmetic, algebra, geometry, and graphs to solve problems.
2. Represent mathematical information and communicate mathematical reasoning symbolically and verbally.
3. Interpret and analyze numerical data, mathematical concepts, and identify patterns to formulate and validate reasoning

Assessment (How do or will students demonstrate achievement of each outcome? Please attach a copy of your assessment electronically.)

The students demonstrate achievement of each outcome by completion of 4 exams, 3 quizzes, and 8 graded assignments, which include projects and problems. I prepare the assessments. I base them on the learning objectives which in turn are aligned with the Student Learning Outcomes. Approximately 75% of the student's grade is based on exams and quizzes. The remaining 25% is based on graded assignments, which include projects and problems.

Starting in the Fall 2010 common problems from the exams, correlated with the learning outcomes have been selected and administered from semester to semester. The results and the common assessment problems are given below

Student Learning Outcome	1	2	3	4 & 6	5
Fall 2010	72%`	68%	62%	74%	70%
Fall 2011	68%	63%	61%	73%	67%
Fall 2012	98%	73%	50%	80%	78%

1. (Student Learning Outcome 1) Suppose A = $\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$. Find A^{-1} and use it to solve $A\mathbf{x} = \mathbf{b}$.
2. (SLO 2) Part A) Describe all solutions of $A\mathbf{x} = \mathbf{0}$ in parametric form, where A is row equivalent to the matrix $\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 3 \end{bmatrix}$.

   Part B) Give a precise geometric description of the solution to Part A.
3. (SLO 3) If A is n x n and the equation $A\mathbf{x} = \mathbf{y}$ has more than one solution for some $\mathbf{y}$ in R^n, can the columns of A span R^n? Explain why or why not.
4. (SLO 4 & 6) Orthogonally diagonalize the following matrix. To save you time, the eigenvalues are 7 and -2.

$$\begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

5. (SLO 5) What exactly does it mean for a vector $\mathbf{w}$ to be in the span$\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$?

Validation (What methods have you used or will you use to validate your assessment?)

The assessments test student achievement of the learning objectives. Validity to a large part is then determined by the appropriateness of the topics and the learning objectives. This is discussed in **My basis for determining the appropriate topics and units for the course** and discussed in **My basis for creating the objectives**, which is appended to the COG. I have also applied the AAAS Categories for Evaluating Instructional Materials to review the curriculum's effectiveness. The analysis is also appended to the COG.

Results (What do your assessment data show? If you have not yet assessed student achievement of your learning outcomes, when is assessment planned?)

After applying the AAAs Categories for Evaluating Instructional Materials I rate the curriculum effectiveness of Linear Algebra as taught at HCC using David Lay's *Linear Algebra* as excellent.

The exams, quizzes and graded assignments directly assess student achievement of the learning objectives. I have not analyzed the grades in general. However, I have modified my teaching to improve scores on questions linked to particular learning objectives. I have not quantified the effectiveness of my modifications.

Results of the common assessment problems indicate a need to improve the critical analysis and construction of mathematical arguments that relate to the study of introductory linear algebra. This result is to be expected since in this introductory 200 level class students are just beginning to formulate proof and reasoning skills. The common assessment results indicate average to above average achievement of the other learning outcomes.

Follow-up (How have you used or how will you use the data to improve student learning?)

(see **Results** section)

Continued development and improvement of logical arguments by the students through the use of written exposition homework and assessment problems.

Budget Justification (What resources are necessary to improve student learning?)

Starting Fall 2013 some assessment questions will come from sources such as Praxis, AP Calculus, GRE subject and SAT subject exams. This will allow results to be benchmarked against a national average. There may be minor budget requirements to acquire test sources.

<u>My basis for determining the appropriate topics and units for the course</u>

This course is offered at Hagerstown Community College (HCC). Many of our students transfer to schools within the Maryland University System. Therefore, we must make sure, for transfer of credits, that our courses are aligned with those in the Maryland University System.

When selecting the text and designing the course I sought the advice of former colleagues at Salisbury University, which is within the Maryland University System. They highly recommended

Linear Algebra, by David Lay of the University of Maryland, College Park. I modeled the topics and the units on the Salisbury University syllabus for Linear Algebra. I made sure that these topics and units included the core topics that were suggested by David Lay, as well as four supplementary topics, and five applied topics.

My goal was to insure that any student taking Linear Algebra at HCC would be well-prepared for any subsequent course work that required Linear Algebra as a prerequisite, even if they transferred outside the Maryland System. After careful study and knowledge acquired at a linear algebra workshop at Drexel University, where the key presenters were Gilbert Strang of MIT and Peter Lax, 2005 Abel Prize Laureate, of NYU, I was confident that the text and topics that I had chosen would accomplish this goal.

David Lay has received one of MAA's Awards for Distinguished College or University Teaching of Mathematics. He was a founding member of the NSF-sponsored Linear Algebra Curriculum Study Group. He is a member of the International Linear Algebra Society and the Society for Industrial and Applied Mathematics. The main goal of the text is to help students master the basic concepts and skills they will need later in their careers. As stated in the preface, "The topics here follow the recommendations of the Linear Algebra Curriculum Study Group, which were based on a careful investigation of the real needs of the students and a consensus among professionals in many disciplines that use Linear Algebra."

In addition, the topics require the use of current technology, with support provided by the text and ancillary materials. Furthermore, the text and the topics incorporate innovations in the theory of linear algebra, particularly with the modern view of matrix multiplication, which is built from the linear combination definition of a matrix-vector product, rather than relying on matrix entries. This simplifies much of the theory for the student and creates a direct transition from linear systems to matrix algebra to vector spaces.

My basis for creating the objectives. How the objectives build on prior knowledge and how they prepare students for later coursework/career goals

My basis for creating the objectives was to make sure the course covers the content as identified by the topics. The objectives were also created with the Student Learning Outcomes in mind. The six learning outcomes are stated in item 6 of the course syllabus. I have linked each learning objective with corresponding student learning outcomes. Students will demonstrate understanding verbally in class and in writing on assignments and exams. Students tend to work collaboratively with each other and sometimes confer with me on the assigned applied problem projects. This is how collaborative skills are demonstrated. The technology required is a graphing calculator or MATLAB software. The technology is necessary to work the applied problems.

This course is an excellent combination of theory and application. The theory does not depend so much on previous knowledge as it does on mathematical maturity. The minimum prerequisite is Calculus II. Mathematical maturity is dependent on the student having a solid conceptual understanding of prerequisite material.

The proofs that the students are responsible for are brief and demonstrate direct connections between important and fundamental concepts that form the foundation of the theory. It is an appropriate setting for students to begin to understand and reproduce simple proofs. Exercises in the problem sets provide students with the opportunity to construct elementary proofs on their own.

Much of the course is an extension and generalization of solving systems of linear equations that students have seen in high school algebra. In this way the course builds directly on prior algebraic knowledge. The geometry and the notation in R^3 and R^n are a natural extension of the geometry and notation that students have seen and used in R^2, the Cartesian Plane. The abstractions and generalizations will help students in subsequent math courses where the spaces they study may consist of vectors whose components are matrices, polynomials or functions, rather than vectors whose components are merely real numbers.

It also should be noted that from prior experience with the Cartesian Plane, students have an intuitive but embryonic conception of the notions of independence, rank, basis, coordinate systems, projection, length and orthogonality. These concepts are formally defined and clarified in this course. In fact, the Cartesian Plane is used to introduce these concepts. Furthermore, a line that passes through the origin in the Cartesian Plane is an example of a Linear Transformation. This concept is also formalized and extended so that the domain and range of a linear transformation consist of vectors.

Because of the work with vectors, R^3, matrices and determinants this course helps in the understanding of Vector and Multivariable Calculus, as well as Differential Equations. The reverse statement is also true. The work with vector spaces will be vital to students who will eventually take Functional Analysis.

Computer science with its use of parallel processing and large-scale computations is directly linked with linear algebra. The topics that are covered provide the foundation for work in such vastly different careers as oil exploration, linear programming and designing electrical networks. In aircraft design the process for finding the airflow around a plane involves solving systems of linear equations. Vector spaces are used in engineering, physics and statistics. For example, input – output signals in an engineering system are functions that have algebraic properties that are like the properties of adding vectors in R^n and multiplying a vector by a scalar. The applications that are presented in the course in the form of projects serve to expose students to a few of the real world uses of the linear algebra theory that they have learned.

Application of the AAAS Categories for Evaluating Instructional Materials to review the curriculum's effectiveness

In the following I will review the effectiveness of the curriculum for Linear Algebra at HCC using the Instructional Analysis from AAAS as applied to Units 1 through 7 for which I developed learning objectives which were linked to student learning outcomes.

Identifying a Sense of Purpose

Each of the units is introduced with a motivating real life problem or situation. For Units 1, 2 and 4 the respective motivations are Linear Models in Economics and Engineering, Computer Models in Aircraft Design, and Space Flight and Control Systems. The motivating examples provide explicit purpose for studying the particular topics within the unit. The learning objectives were then determined to insure the student achieves the appropriate student learning outcome while mastering the content. For example, to construct a linear model that would have application to economics the motivating example shows that it is necessary to solve systems of linear equations. This becomes the reason for the first topic in Unit 1. To gain mastery in solving systems of linear equations the student must manipulate equivalent systems. Therefore, one of the objectives is that the student understands what it means for linear systems to be equivalent. This objective is linked with learning outcomes 4 and 5.

Building on Student Ideas

The material develops directly from a student's knowledge of high school algebra and their understanding of the Cartesian Plane. The text implicitly makes these connections in the problem sets, but I find that I sometimes have to make the connections explicit to the students during lecture. It might be that the author sometimes assumes that these connections are so elementary that they don't need to be made explicit. However, on the other hand, I must admit that the text makes clever use of graphics that I wouldn't have thought of to illustrate some of the ideas. As an example, I think the lattice graphs the author introduces to plot points when working with unfamiliar bases are especially revealing. They place the strangeness of working in a different coordinate system in the familiar context of the Cartesian Plane with the standard coordinate system. The geometric and conceptual utility of this device is the motivation for the learning objective *Be able to give a graphical interpretation of coordinates* in Unit 4, Topic 5.

Engaging Students

Each unit ends with sections of real life, applied problems for the students to solve. This provides a direct connection to the motivating example that introduced the unit and allows the students to apply in a realistic setting the knowledge and skills that were developed by mastery of the unit. There are also a variety of project problems to select from. Typically, there is only enough time in the course to assign four project problems. I vary these so students will see the range of application of the course content. I assign project problems from social science, economics, science, and engineering.

Developing Ideas

The text does an excellent job of connecting ideas. As I have mentioned earlier, the text incorporates innovations in the theory of linear algebra, particularly with the modern view of matrix multiplication, which is built from the linear combination definition of matrix–vector product. This simplifies much of the theory, dispensing with a lot of cumbersome notation, and allows for a direct transition from linear systems to matrix algebra to vector spaces. The importance of the modern view of matrix multiplication is the motivation for the learning objective *Understand and be able to apply the linear combination definition of* $A\mathbf{x}$ of Unit 1, Topic 3. It is also the motivation for the learning objectives *Understand and be able to apply the algebraic operations and properties of matrices* and *Understand and be able to apply the definitions of matrix multiplication that corresponds to composition of linear transformations* of Unit 2, Topic 1.

The development of the Invertible Matrix Theorem (IVT) is another example of how well the text develops ideas. IVT is first introduced in Unit 2 with 12 equivalent statements. IVT is further developed as new topics are introduced so that by the time Unit 5 is reached IVT has been expanded to 20 equivalent statements. These 20 statements provide logical connections from material learned at the beginning of the course to topics considered at the end.

In the section *Identifying a Sense of Purpose* I have already explained how the text provides applications of knowledge. The text also provides ample practice opportunities. For example, in the problem set related to Unit 4, Topic 6, the introductory problems are computational and algebraic in nature, where students are asked to find the basis and state the dimension. These types of problems link to Student Learning Outcome 1. Later in the exercise set are of the types that require proof and reasoning. For example, under stated conditions students are asked to prove that $\dim T(H) \leq \dim H$. These types of problems are related to Student Learning Outcome 3. Some of the problems require written explanations and involve interpretive skills. Students are asked to explain why S can not span *V*. They are also asked to explain why the space of all polynomials is an infinite dimensional space. These types of problems are connected to Student Learning Outcome 5. Two problems in the set have the appropriate icon to alert students that technology is required. These problems are linked to Student Learning Outcome 4.

Additionally, there are problems that require prior knowledge from prerequisite courses, such as calculus. For example, students are asked to show that the space of all continuous functions defined on the real number line is an infinite-dimensional space. Continuous functions are studied in Calculus I. Finally, there are problems introducing and using Hermite and Laguerre polynomials. This provides a connection to subsequent work in math, particularly to the study of differential equations.

Promoting Student Thinking

I have already provided specific examples in the previous section, *Developing Ideas*, that show how Student Learning Outcomes 3 and 5 are involved in the course. In addition, in the same unit (Unit 4), students use visualization and spatial reasoning to understand and solve problems that involve coordinate mappings. Thus all Student Learning Outcomes, numbers 2, 3 and 5, that promote student thinking are incorporated in this particular unit. Upon examination, this is the case for all other units.

The *Study Guide Linear Algebra and Its Applications* is a course requirement (see item 5 of the syllabus). The study guide is designed to promote student thinking. Some problems in the guide are worked in detail. Others are left incomplete, but with helpful solution tips or hints. It should also be mentioned that the study guide addresses misconceptions. One of the most common misconceptions appears as a bold warning to alert students to the fact that $(AB)^{-1} = B^{-1}A^{-1}$ only if both A and B are invertible.

The study guide also shows students how to study mathematics and prepare review sheets for tests. These organizational strategies also serve to develop student thinking.

Assessing Student Progress

I prepare the assessments. I base them on the learning objectives which in turn, as I have demonstrated in the section *Units and Topics*, are aligned with the Student Learning Outcomes. Approximately 75% of the student's grade is based on exams and quizzes. The remaining 25% is based on projects. The projects serve to provide variety to the assessments. Thus, if a student does not perform well on tests, he/she can still earn a high grade through superior work on assigned projects.

Enhancing the Learning Environment

The following instructional aids provide student and teacher content support:

Transparency Masters With examples and fill-in-the-blank problems supports all topics.
The Student Study Guide Is designed to teach students how to learn mathematics.
Technology Manuals Are written specifically for the text. They include class-tested projects ready for use.
Interactive Maple Modules That can be used as geometric classroom demonstration. Others can be used as labs for individual or collaborative work.
MyMathLab Can be accessed online and contains links to related material, review sheets, sample exams and solutions. It also has a complete course management system.

Math Tutor Center Offers live tutoring from qualified instructors.
Instructor's Solution Manual Contains detailed solutions to all problems.

Conclusion

After applying the AAAS Categories for Evaluating Instructional Materials I would rate the curriculum effectiveness for Linear Algebra as taught at HCC using David Lay's *Linear Algebra* as excellent..

Course: MAT 208 SLOA Data

Faculty Team

	FA 2010	**FA 2011**	**SP 2012**	**FA 2012**
# Active students	11	13	1	20
%W	18.2	7.7	0	0
*% walk-away Fs No final exam/grade = F	18.2	0	0	20
% Success (A,B,C)	54.5	75	100	75
Common Comprehensive Final Exam Score	69.2	66.8	87	75.8
Mean course grade	2.0	2.55	4.0	
Item Analysis **Weakest Content Areas**	SLO 3	SLO 3		SLO 3

*% Walk-away Fs = Did not take the final exam and received a grade of F.