

Name:

Score (for office use):

SMMART Team Selection Test
Hagerstown Community College: STEM Club

Instructions.

Complete as many problems as possible and show all of your work in the space provided. The intention is that you spend 60 minutes (1 hour) to complete the five problems listed. If you take longer, make a note of how long it took you on this page.

This is due by the end of the work day on Friday, February 12. Submit your solutions to Joseph Heavner via email (see paragraph 3), in person, or by leaving it on his desk in the Student Activities Office in the Student Center (STC-165).

Do not reference answers online, or cheat in any other way. Your work will be checked for plagiarism, and if an individual is found guilty of academic dishonest, (s)he will be dismissed from the team immediately. You may use references if you forget a definition or theorem, and you may also ask the administrator of the exam for any clarification (email: jrheavner@student.hagerstowncc.edu). Cite all sources used on this front page, even if it is something as benign as using Wikipedia to remember the definition of $\text{lcm}(a, b)$.

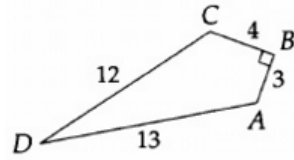
The grading criteria is mostly based on correctness and work showing the legitimacy of the solution. In the case of an incorrect or incomplete response, the work will be graded and partial credit awarded where appropriate. Answers without any work will receive no credit. Reasonable skips in solution work will not be penalized (*e.g.* skipping algebraic steps to solve a quadratic equation). Excessive use of external resources will result in a minor loss of points, as will excessive use of time. Proofs are not required due to the time constraint.

You may *only* use a graphing calculator, a writing utensil, and scratch paper.

The top three scorers will comprise HCC's SMMART team. At this time, there will be only one team, though this may change. Other participants will be given the option to become a reserve to fill the spot of another in case of absence, or to help with other competition needs. SMMART will take place on Saturday, April 16, 2016, in the Student Center. Registration begins at 9:30 a.m..

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1. Sides AB , BC , CD , and DA of the convex quadrilateral $ABCD$ have lengths 3, 4, 12, and 13, respectively; and $\angle CBA$ is a right angle. What is the area of quadrilateral $ABCD$?



2. Write $\text{lcm}(a, b)$ (“Least Common Multiple”) , for a, b integers, in terms of $\text{gcd}(a, b)$ (“Greatest Common Divisor”).

Hint: Try cases using some pairs that share factors and some that don't (coprime).

3. If $\frac{1}{a+c} = \frac{1}{a} + \frac{1}{c}$, find $(a/c)^3$. (Find the numerical value.)

Hint: Consider the identity $(a^2 + ac + c^2)(a - c) = a^3 - c^3$.

4. How many scalene triangles have all sides of integer length and perimeter less than 13?

5. Find the largest n such that 2^n divides $100!$

Note: $!$ denotes the factorial function, defined for a positive integer n as:

$$n! = n \cdot (n - 1) \cdot (n - 2) \cdots 3 \cdot 2 \cdot 1$$